

CP Violation in Two Zero Texture Neutrino Mass Matrices

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Abstract

It has been shown that the neutrino mass matrices with two texture zeros in the charged lepton basis predict non-zero 1-3 mixing and are necessarily CP violating with one possible exception in class C for maximal mixing.

There is no CP violation in the leptonic sector of the Standard Model (SM) of fundamental particles and interactions. However, in most extensions of SM, there can be several CP phases. In the simplest three generation scenario, there is a Dirac type CP violating phase in the leptonic mixing matrix. However, for Majorana neutrinos, there could be two additional phases (Majorana phases). It is possible to work in a parameterizations in which all the three CP violating phases are situated in the charged current leptonic mixing matrix. Without any loss of generality, one can work in the flavor basis in which the charged lepton mass matrix is diagonal so that the neutrino mass matrix carries all the information about CP violation. In the flavor basis, the mass matrix for Majorana neutrinos contains nine physical parameters including the three mass eigenvalues, three mixing angles and the three CP-violating phases. The two squared-mass differences (Δm_{12}^2 and Δm_{13}^2) and the two mixing angles (θ_{12} and θ_{23}) have been measured in solar, atmospheric and reactor experiments. The third mixing angle θ_{13} and the Dirac-type CP-violating phase δ are expected to be measured in the forthcoming neutrino oscillation experiments. The possible measurement of the effective Majorana mass in neutrinoless double β decay searches will provide an additional constraint on the remaining three neutrino parameters viz. the neutrino mass scale and two Majorana-type CP-violating phases. While the neutrino mass scale will be independently determined by the direct beta decay searches and cosmological observations, the two Majorana phases will not be

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uniquely determined from the measurement of effective Majorana mass even if the absolute neutrino mass scale is known. Under the circumstances, it is natural to employ other theoretical inputs for the reconstruction of the neutrino mass matrix. The possible forms of these additional theoretical inputs are limited by the existing neutrino data. Several proposals have been made in the literature to restrict the form of the neutrino mass matrix and to reduce the number of free parameters which include presence of texture zeros [1, 2, 3, 4, 5], requirement of zero determinant [6], the zero trace condition [7] to name just a few. However, the current neutrino oscillation data are consistent only with a limited number of texture schemes [1, 2]. In particular, the current neutrino oscillation data disallow all neutrino mass matrices with three or more texture zeros in the flavor basis. Out of the fifteen possible neutrino mass matrices with two texture zeros, only seven are compatible with the current neutrino oscillation data. The seven allowed two texture zero mass matrices have been classified into three categories. The two class A matrices of the types A_1 and A_2 give normal hierarchy of neutrino masses. The class B matrices of types B_1 , B_2 , B_3 and B_4 yield a quasi-degenerate spectrum of neutrino masses. The single class C matrix corresponds to inverted hierarchy of neutrino masses.

The texture zeros in different positions of the neutrino mass matrix can result from underlying flavor symmetries [8]. They can, also, be realized within the framework of the see-saw mechanism [9]. Such textures can, also, be obtained in the context of GUTs based on $SO(10)$ [10, 11]. Recently, these texture zeros have been derived from a simple flavor group A_4 or its Z_3 subgroup [12].

The existence of two texture zeros implies four real conditions to be satisfied by the neutrino oscillation parameters. It is shown that these four conditions cannot be satisfied if $\theta_{13} = 0$. The presence of texture zeros in the neutrino mass matrix, further, demands certain cancellations induced by strong correlations between CP-violating phases. It is shown that neutrino mass matrices with two texture zeros in the charged lepton basis must be necessarily CP-violating with one possible exception in class C for $\theta_{23} = \frac{\pi}{4}$ and $\theta_{13} = 0$ where there will be no Dirac-type CP-violation and we can, only, have Majorana-type CP-violation. This case can become CP conserving for a special choice of m_1 .

The neutrino mass matrix, M , can be parameterized in terms of the three neutrino mass eigenvalues (m_1, m_2, m_3), three neutrino mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and one Dirac-type CP violating phase, δ . If neutrinos are Majorana particles, then, there are two additional CP violating phases α, β in the neutrino mixing matrix. The complex symmetric mass matrix M can be diagonalized by a complex unitary matrix V :

$$M = V M_\nu^{diag} V^T \quad (1)$$

where $M_\nu^{diag} = \text{Diag}\{m_1, m_2, m_3\}$. The neutrino mixing matrix V can be written as

$$V \equiv UP = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i(\beta+\delta)} \end{pmatrix}, \quad (2)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The matrix V is called the neutrino mixing matrix or Pontecorvo-Maki-Nakagawa-Sakata matrix. The matrix U is the lepton analogue of

Type	Constraining Equations
A_1	$M_{ee} = 0, M_{e\mu} = 0$
A_2	$M_{ee} = 0, M_{e\tau} = 0$
B_1	$M_{e\tau} = 0, M_{\mu\mu} = 0$
B_2	$M_{e\mu} = 0, M_{\tau\tau} = 0$
B_3	$M_{e\mu} = 0, M_{\mu\mu} = 0$
B_4	$M_{e\tau} = 0, M_{\tau\tau} = 0$
C	$M_{\mu\mu} = 0, M_{\tau\tau} = 0$

Table 1: Allowed two texture zero mass matrices.

the Cabibbo-Kobayashi-Maskawa quark mixing matrix and P contains the two Majorana phases.

The elements of the neutrino mass matrix can be calculated from Eq. (1). Some of the elements of M , which are equated to zero in the various allowed texture zero schemes, are given by

$$M_{ee} = c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{2i\alpha} + s_{13}^2 m_3 e^{2i\beta}, \quad (3)$$

$$M_{e\mu} = c_{13} \{ s_{13} s_{23} e^{i\delta} (e^{2i\beta} m_3 - s_{12}^2 e^{2i\alpha} m_2) - c_{12} c_{23} s_{12} (m_1 - e^{2i\alpha} m_2) - c_{12}^2 s_{13} s_{23} e^{i\delta} m_1 \}, \quad (4)$$

$$M_{e\tau} = c_{13} \{ s_{13} c_{23} e^{i\delta} (e^{2i\beta} m_3 - s_{12}^2 e^{2i\alpha} m_2) + c_{12} s_{23} s_{12} (m_1 - e^{2i\alpha} m_2) - c_{12}^2 s_{13} c_{23} e^{i\delta} m_1 \}, \quad (5)$$

$$M_{\mu\mu} = m_1 (c_{23} s_{12} + e^{i\delta} c_{12} s_{13} s_{23})^2 + e^{2i\alpha} m_2 (c_{12} c_{23} - e^{i\delta} s_{12} s_{13} s_{23})^2 + e^{2i(\beta+\delta)} m_3 c_{13}^2 s_{23}^2 \quad (6)$$

and

$$M_{\tau\tau} = m_1 (s_{23} s_{12} - e^{i\delta} c_{12} s_{13} c_{23})^2 + e^{2i\alpha} m_2 (c_{12} s_{23} + e^{i\delta} s_{12} s_{13} c_{23})^2 + e^{2i(\beta+\delta)} m_3 c_{13}^2 c_{23}^2. \quad (7)$$

The seven allowed neutrino mass matrices with two texture zeros have been listed in Table 1.

For neutrino mass matrices of type A_1 , the conditions $M_{ee} = 0$ and $M_{e\mu} = 0$ imply [4]

$$s_{13}^2 (m_1 c_{12}^2 + m_2 s_{12}^2 \cos 2\alpha - m_3 \cos 2\beta) = c_{12}^2 m_1 + s_{12}^2 m_2 \cos 2\alpha, \quad (8)$$

$$s_{13}^2 (m_2 s_{12}^2 \sin 2\alpha - m_3 \sin 2\beta) = s_{12}^2 m_2 \sin 2\alpha, \quad (9)$$

$$s_{13} (m_1 c_{12}^2 \cos \delta + m_2 s_{12}^2 \cos(2\alpha + \delta) - m_3 \cos(2\beta + \delta)) = \frac{c_{12} c_{23} s_{12} (m_2 \cos 2\alpha - m_1)}{s_{23}}, \quad (10)$$

$$s_{13} (m_1 c_{12}^2 \sin \delta + m_2 s_{12}^2 \sin(2\alpha + \delta) - m_3 \sin(2\beta + \delta)) = \frac{m_2 c_{12} c_{23} s_{12} \sin 2\alpha}{s_{23}}. \quad (11)$$

It can be seen from Eqs. (9) and (11) that $\sin 2\alpha$ must vanish if s_{13} is zero. However, the simultaneous solution of Eqs. (8) and (10) under the condition $s_{13} = 0$ implies that $m_1 = m_2 = 0$ which is inconsistent with solar neutrino data. Therefore, s_{13} should be non-zero for neutrino mass matrices of type A_1 [3, 4, 5] which in turn implies a non-vanishing $\sin 2\alpha$ [Eqs. (9) and (11)]. It can, also, be seen from Eqs. (9) and (11) that $\sin 2\alpha = \sin 2\beta = 0$ if $\sin \delta = 0$ which is not possible for non-zero s_{13} . Similarly, one

can show that s_{13} and $\sin \delta$ are non-zero for neutrino mass matrices of type A_2 . Hence, neutrino mass matrices of class A are necessarily CP-violating [4].

In category B, for neutrino mass matrices of type B_3 , for example, we have the conditions $M_{e\mu} = 0$ and $M_{\mu\mu} = 0$ which result in the following four real conditions:

$$s_{13}(m_1 c_{12}^2 \cos \delta + m_2 s_{12}^2 \cos(2\alpha + \delta) - m_3 \cos(2\beta + \delta)) = \frac{c_{12} c_{23} s_{12}(m_2 \cos 2\alpha - m_1)}{s_{23}}, \quad (12)$$

$$s_{13}(m_1 c_{12}^2 \sin \delta + m_2 s_{12}^2 \sin(2\alpha + \delta) - m_3 \sin(2\beta + \delta)) = \frac{m_2 c_{12} c_{23} s_{12} \sin 2\alpha}{s_{23}}, \quad (13)$$

$$s_{13}(m_2 \cos(2\alpha + \delta) - m_1 \cos \delta) = \frac{c_{23}^2(m_1 s_{12}^2 + m_2 c_{12}^2 \cos 2\alpha) + m_3 s_{23}^2 \cos 2(\beta + \delta)}{c_{12} c_{23} s_{12} s_{23}} \quad (14)$$

and

$$s_{13}(m_2 \sin(2\alpha + \delta) - m_1 \sin \delta) = \frac{m_2 c_{23}^2 c_{12}^2 \sin 2\alpha + m_3 s_{23}^2 \sin 2(\beta + \delta)}{c_{12} c_{23} s_{12} s_{23}}. \quad (15)$$

It can be easily seen from Eqs. (12) and (13) that $m_1 = m_2$ and $\sin 2\alpha = 0$ for $\theta_{13} = 0$. Hence, θ_{13} and α cannot be zero since solar mass squared difference Δm_{12}^2 is non-zero. It follows from Eqs. (13) and (15) that $\sin 2\alpha$ and $\sin 2\beta$ are zero if $\sin \delta = 0$ which is not possible since s_{13} is non-zero. Therefore, $\sin \delta$ should be non-zero for neutrino mass matrices of type B_3 . Similarly, one can show that θ_{13} and $\sin \delta$ are non-zero for other neutrino mass matrices of class B viz. B_1 , B_2 and B_4 . Thus, neutrino mass matrices of class B are, also, necessarily CP violating.

The four real conditions to be satisfied by neutrino mass matrices of class C can be written as

$$\begin{aligned} s_{13}^2(m_3 \cos 2(\beta + \delta) - m_1 c_{12}^2 \cos 2\delta - m_2 s_{12}^2 \cos 2(\alpha + \delta)) \\ = m_1 s_{12}^2 + m_2 c_{12}^2 \cos 2\alpha + m_3 \cos 2(\beta + \delta), \end{aligned} \quad (16)$$

$$\begin{aligned} s_{13}^2(m_3 \sin 2(\beta + \delta) - m_1 c_{12}^2 \sin 2\delta - m_2 s_{12}^2 \sin 2(\alpha + \delta)) \\ = m_2 c_{12}^2 \sin 2\alpha + m_3 \sin 2(\beta + \delta), \end{aligned} \quad (17)$$

$$\sin 2\theta_{12}(m_1 \cos \delta - m_2 \cos(2\alpha + \delta))s_{13} = -2(m_1 s_{12}^2 + m_2 c_{12}^2 \cos 2\alpha) \cot 2\theta_{23} \quad (18)$$

and

$$\tan \theta_{12}(m_1 \sin \delta - m_2 \sin(2\alpha + \delta))s_{13} = -m_2 \sin 2\alpha \cot 2\theta_{23}. \quad (19)$$

When, θ_{23} is non-maximal, the condition $s_{13} = 0$ gives $\sin 2\alpha = 0$ and $\frac{m_2}{m_1} = \tan^2 \theta_{12} < 1$ from Eqs. (18) and (19) which contradicts the solar mass hierarchy. It can, also, be shown that the above system of equations cannot simultaneously hold for $\sin \delta = 0$ since Eq. (19) implies that $\sin 2\alpha = 0$ under this condition which, again, gives $s_{13} = 0$ which, as discussed above, is inconsistent with the data. Therefore, the neutrino mass matrices of class C are necessarily CP violating for non-maximal 2-3 mixing.

For maximal mixing, $\theta_{23} = \frac{\pi}{4}$ and the Eqs. (18) and (19) imply that $\theta_{13} = 0$. In this case, there will be no Dirac-type CP violation since $s_{13} = 0$ and we are left with the Eqs. (16) and (17) only which can be written as

$$s_{12}^2 m_1 + c_{12}^2 m_2 \cos 2\alpha = -m_3 \cos 2\beta, \quad (20)$$

and

$$c_{12}^2 m_2 \sin 2\alpha = -m_3 \sin 2\beta. \quad (21)$$

From Eqs. (20) and (21), the only possible CP conserving solution for neutrino mass matrices of class C with maximal mixing is given by

$$\alpha = \frac{\pi}{2}, \beta = 0 \quad (22)$$

and

$$m_1^2 = \frac{1}{\sin^2 2\theta_{12}} \frac{(\Delta m_{12}^2 c_{12}^4 - \Delta m_{13}^2)^2}{\Delta m_{12}^2 c_{12}^2 - \Delta m_{13}^2}. \quad (23)$$

This solution is valid only for inverted hierarchy with Δm_{13}^2 negative. However, it may be difficult to realize the relationship given in Eq. (23) in a realistic model of lepton masses and mixings. In this special CP conserving case, the CP parity of ν_2 is odd and that of ν_3 is even. CP is conserved in the region allowed by Eqs. (22) and (23) within the complete parameter space allowed by Eqs. (20) and (21) and in the rest of the allowed parameter space, the neutrino mass matrices of class C with maximal mixing will have CP violation of Majorana type only. This is a consequence of the fact that the third neutrino mass eigenstate ν_3 gets decoupled from the first two mass eigenstates ν_1 and ν_2 for vanishing s_{13} and the remaining 2×2 sub-matrix can have only Majorana-type CP violation with a single physical Majorana phase. Hence, the neutrino mass matrices of class C with maximal mixing can be CP conserving only for m_1 given by Eq. (23). For all other values of m_1 , the neutrino mass matrices of class C with maximal mixing will exhibit CP violation with one physical Majorana phase.

It should, also, be noted that the neutrino mass matrices of class C with maximal mixing are $\mu - \tau$ symmetric since we have

$$M_{e\mu} = -M_{e\tau} \text{ and } M_{\mu\mu} = M_{\tau\tau} = 0 \quad (24)$$

in this special case. However, the neutrino mass matrices of classes A and B can never become $\mu - \tau$ symmetric which results in a different CP structure of class C as compared to that of classes A and B of neutrino mass matrices.

In conclusion, it has been shown that CP violation is inherent in the two texture zero scheme of Frampton, Glashow and Marfatia [1] and existence of texture zeros in the neutrino mass matrix may provide useful hints for unraveling the dynamics of the CP violation. The existence of the texture zeros in the neutrino mass matrix in the flavor basis could result from a certain flavor symmetry. This, in turn, would require non-zero values of the mixing angle θ_{13} and the CP violating phases to ensure the desired cancellations in the specific elements of the neutrino mass matrix.

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